Preface

Systems of many interacting particles or degrees of freedom respond in complex ways when driven out of equilibrium If the departure from equilibrium is small linear response theory can be used to study the properties of the system for such a description knowledge of the equilibrium state is sufficient If the driving forces are large however the steady state of the system usually cannot be understood in terms of its equilibrium properties Examples of such steady states include fluid turbulence and flows in granular media In addition it has been proposed recently that the dynamics of a vast variety of physical systems coupled to external environments is such that they evolve naturally to states whose response to perturbations is scale-invariant in both space and time It is conjectured that such self organization underlies the wealth of fractal structures seen in nature

The study of the complete set of equations which govern these steady states (such as master equations for lattice systems or the coarse-grained hydrodynamic equations describ ing the conservation of number momentum and energy densities in a fluid) is typically a formidable task For this reason it is useful first to study simple models which retain as many essential features of the problem as possible In this thesis we study the detailed dy namical behaviours of two such simple model systems *viz* a driven version of the sandpile model (originally proposed in the context of *Self Organized Criticality*) and the GOY shell model for homogeneous isotropic fluid turbulence Though these models are simple they generate extremely complex dynamics which we explore in detail

This thesis is organized as follows

The concept of *Self Organized Criticality* (SOC) was introduced to explain the occurrence of generic scale-invariance in a variety of natural systems *Sandpile models* were initially proposed as the simplest models that display SOC In Chapter 1 we briefly introduce these models starting with the original one proposed by Bak Tang and Wiesenfeld We then discuss some numerical studies of universality in sandpile models Some exact calculations on a class of sandpile models are discussed briefly as are some related models which also show SOC The roles of conservation laws and anisotropy in producing SOC in a system of stochastic partial differential equations have been analyzed by some authors these are summarized Finally we turn our attention to the critical behaviours of driven versions of sandpile models studied previously We conclude this Chapter with a discussion of some of the unanswered questions in this area

In Chapter 2 we study the possible large-scale behaviours of a one-dimensional *nearest neighbour* hopping sandpile *slope* model with non conserving noise via an exact decimation procedure This work is a contribution to the effort of classifying sandpile models into *universality classes* We consider one-dimensional sandpile models in which the height (a real continuous variable) at each site can change either by a random addition or subtraction (non conserving noise) or by a transfer to or from a neighbouring site The magnitude of the transfer current depends on the local height gradient only The simple structure of our models allows us to make exact statements about their possible long time large-scale behaviour via an exact decimation procedure We map these models onto dynamical problems of sets of classical variables at thermal equilibrium governed by Hamiltonians The possible behaviours of the equal time correlation functions at large spatial scales are obtained by classifying the fixed points of an exact decimation procedure applied to the probability distributions of heights Our results show that there is a continuous infinity of fixed points parameterized by an index $n (1 \leq n \leq 2)$ where $n = 2$ gives a conventionally rough one-dimensional interface The models with $n < 2$ show a rougher surface which are significant as new universality classes although they are hard to interpret physically It would be interesting to find physical systems to which they correspond

Chapter 3 describes the results of our study of a driven *critical slope* sandpile model that we define It has a mean input current j in which the current slope relation for slopes exceeding a threshold is controlled by a parameter α Our update rule conserves the particle number locally and generates a local current that increases with local height differences thus preventing an unbounded buildup of particles for large j Mean field arguments enable us to show that the small and large-j_n asymptotes of the steady states have mean slopes $\sigma_a = \sigma$ and $\sigma_a = j_l / 2\alpha$ respectively with σ being the angle of repose Our numerical studies of this model reveal that σ_{av} interpolates between the small and large-j asymptotes via an infinite series of continuous transitions marked by divergences in the equal time height correlation length and the output current autocorrelation time The existence of such phase transitions in a *one dimensional stochastic model with local interactions* is to be contrasted with the situation in the equilibrium case where it is well known that such transitions cannot occur at finite temperature The first of these transitions occurs at $j_l \simeq 0\ 5$ where $\sigma_{av} - \sigma_c$ rises continuously from zero with an exponent $\beta \simeq 0$ ɔ We also monitor the steady states of our model as a function of α and the mean input current j_n and find a rich nonequilibrium phase diagram which shows many distinct phases characterized by the mean slope σ_{av} Many of the features of this phase diagram can be explained through an extension of our mean field theory However in the $j \to 0$ limit this model does not show SOC as indicated by the absence of power law distributions in our numerical results in this limit We also study a two-dimensional version of this model Our results in this case yield continuous transitions as in the one-dimensional case

In Chapter 4 we present a brief overview of theories and experimental results on homogeneous isotropic turbulence We describe the Navier Stokes equations for an incompressible three-dimensional fluid comment on the studies on turbulent flow based on such equations and introduce the concept of homogeneous and isotropic turbulence We then describe phenomenological theories including the 1941 theory of Kolmogorov which we describe in detail Experimental data on turbulence show multiscaling corrections to this theory We describe some recent phenomenological attempts to account for this discrepancy We then introduce the simplified shell models numerical studies of which have over the past few years led to some new insights on turbulence We concentrate on the GOY shell model and discuss the results of earlier numerical studies thereof

In Chapter 5 we describe the details of our numerical study of the GOY shell model We study the asymptotic behaviours of velocity structure functions in both the inertial and the dissipation ranges Our study verifies the scaling property of the energy spectrum and the dependence of Taylor microscale Reynolds number on viscosity The intermittency in the velocity field is characterized by the multiscaling exponents in the inertial range we calculate the variation of these exponents as function of the Reynolds number and show that the deviations of these exponents from their Kolmogorov values *do not* decrease as the Reynolds number is increased We also extend the Generalized Extended Self similarity ansatz for the GOY shell model and broaden its applicability to reveal new scaling behaviour in the dissipation range The crossover from inertial to dissipation range asymptotics is found to be universal as we show by comparing our shell model data with data from a Direct Numerical Simulation of the three-dimensional Navier Stokes equation We also study the probability distribution functions of the velocity fields in the GOY shell model and characterize the crossover from Gaussian to non Gaussian distribution with increasing wavenumber or Reynolds number